Sec 1.2/1.3 (Blend)

* Read example about "terminal velocity" in the [L2-3] Notes.

Newton's Law of Cooling :

T = object's temperature

t = time

A = ambient temperature

NLC : $\frac{dT}{dt} \propto (A-T) \longrightarrow \frac{dT}{dt} = k(A-T)$ ($k>0$ const)

If $T<A \rightarrow T'>0$

$T=A \rightarrow T'=0$.

$T>A \rightarrow T'<0$ (general slope field $y'=f(x,y)$)

If an ODE is $\{y'=f(x)\}$, no y, y', etc then general solution is

$$y = \int f(x)dx + C$$

Ex : Find a general solution for $\{y'=2x+3\}$

$$\rightarrow \frac{dy}{dx} = 2x+3$$

$$\int dy = \int (2x+3)\,dx$$

$$y + c_1 = x^2 + 3x + c_2$$

$$y = x^2 + 3x + \boxed{c_2 - c_1} \quad \text{"C"}$$

$y = x^2 + 3x + C$ is gen. solution.

Now find a particular solution with $y(1) = 2$

plug in $(x=1, y=2)$

$$y = x^2 + 3x + C$$

$$2 = \cancel{x}1 + 3 + C \implies C = -2$$

so particular solution is $\underline{y = x^2 + 3x - 2}$.

"particular solution" of an ODE = "solution to IVP"

$$\left(\begin{array}{l}\text{solution of } y' = 2x+3 \\ \text{having } y(1) = 2\end{array}\right) = \left(\text{solution of IVP} \begin{Bmatrix} y' = 2x+3 \\ y(1) = 2 \end{Bmatrix}\right)$$

Ex: Find a general solution of $\left\{ y' = \dfrac{12}{x^2+16} \right\}$

Recall $\dfrac{d}{dx}\left[\tan^{-1}\left(\dfrac{x}{a}\right)\right] = \dfrac{1}{(x/a)^2+1} \cdot (1/a) = \dfrac{a}{x^2+a^2}$

$$\frac{dy}{dx} = \frac{12}{x^2+4^2} \longrightarrow \int dy = \int 3 \cdot \left(\frac{4}{x^2+4^2}\right) dx$$

$$y + c_1 = 3\left(\tan^{-1}(x/4) + c_2\right)$$

$$y = 3\tan^{-1}(x/4) + \boxed{3c_2 - c_1} \quad \text{"}C\text{"}$$

$$y = 3\tan^{-1}(x/4) + C \quad \text{gen. solution.}$$

Find a particular solution with $y(0) = 1$.

$$1 = 3\tan^{-1}(0/4) + C \longrightarrow 1 = C$$

$$y = 3\tan^{-1}(x/4) + 1 \quad \text{particular solution.}$$

Ex: Acceleration/ Velocity / Position.

→ constant acceleration Example.

Find a general solution for $x(t)$ if

$$\{a(t) = x''(t) = 7\}$$

$$x'' = 7 \implies \underline{x'} = 7t + c_1$$
$$v(t) = 7t + c_1$$

integrating again,

$$x' = 7t + c_1 \implies x = \frac{7}{2}t^2 + \underline{c_1}\,t + \underline{c_2} \quad \text{(gen sol)}$$

★ notice original ODE having 2nd order derivatives means general solution has 2 unknown constants.

$$v(t) = 7t + c_1 \implies v(0) = 0 + c_1 \implies v_0 = c_1$$
(initial velocity)

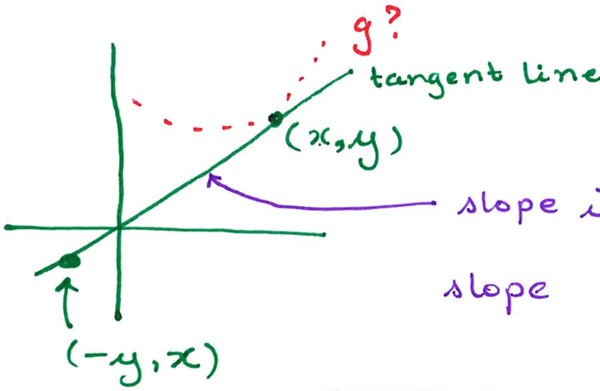
$$\boxed{v(t) = 7t + v_0}$$

similarly $c_2 = x(0) = x_0$ (initial position)

gen solution can be written

$$x(t) = \frac{7}{2}t^2 + v_0 t + x_0$$

HW Example: Write an ODE for graph $y = g(x)$ - such that tangent line at (x,y) passes through $(-y,x)$.



slope is g', $= ?$

slope $\dfrac{y-x}{x-(-y)} = \dfrac{y-x}{x+y}$.

Eq of line generally is $(y-y_0) = m(x-x_0)$

(x_0, y_0) must be on line. $\quad m \uparrow$ slope

~~point on line is~~ Don't need...

Know slope of tangent line at (x,y) is $\dfrac{y-x}{x+y}$.

so then $\dfrac{dy}{dx} = \dfrac{y-x}{x+y}$